Curso: Topics on Geometric Integration for Partial Differential Equations.

Cada ponente impartirá una conferencia sobre el tema (los días 16 y 17 de septiembre de 2025, en horario de 16:00 a 18:00). Las charlas se complementarán con seminarios de discusión, más informales y abiertos a cualquier investigador de la Uva interesado.

Los contenidos del curso son los siguientes:

1. Cipriano Escalante (U. of Málaga)

   Título: Recent advances in the numerical modelling of non-hydrostatic multilayer-moment models.
   Resumen:
   A general framework based on the multilayer-moment technique is presented. This approach leads to first-order derivative non-hydrostatic pressure models with arbitrarily high accuracy in the dispersion diagram. Some robust classical numerical schemes are presented, focusing on projection-correction methods that involve solving an incompressibility condition, as well as techniques that relax this constraint.

2. David Ketcheson (KAUST)

   Título: Asymptotic preserving discretizations for hyperbolic approximations of dispersive wave equations
   Resumen:
   I will describe a technique for constructing hyperbolic PDE approximations of higher order dispersive wave equations like the KdV, BBM, and NLS equations. These hyperbolic systems take the form of a relaxation system that formally approximates the original problem in the limit as the relaxation parameter vanishes. Then I will show how IMEX Runge-Kutta methods can be used to efficiently integrate these systems in a way that also ensures they numerically approximate the original problem when the relaxation parameter is small.

3. Hendrik Ranocha (JGU Mainz)

   Título: Using summation-by-parts (SBP) operators to construct structure-preserving discretizations
   Resumen:
   Partial differential equations (PDEs) can have several geometric structures. Structure-preserving methods can provide both qualitative and quantitative enhancements to numerical solutions, such as ensuring stability and robustness, as well as improving accuracy. In many instances, PDEs are linked to functionals of the solution that are either conserved or dissipated. For instance, the total energy of a nonlinear wave equation is typically conserved, depending on the boundary conditions. In this talk, we will review how summation-by-parts (SBP) operators can be used to construct structure-preserving discretizations of PDEs. SBP operators provide discrete derivative and integration operators that mimic integration by parts. When combined with split forms of the PDEs, SBP operators can be used to obtain energy-conserving or entropy-stable discretizations of linear and nonlinear PDEs. We present several examples and highlight how SBP operators enable a unified analysis of structure-preserving properties in finite difference, finite volume, and (pseudo-)spectral methods collocation, continuous finite element, and discontinuous Galerkin methods.

4. Mario Richiutto (INRIA)

Título: Steady state preservation, well balanced and error balance for hyperbolic equations: from consistency with constants to the preservation of differential operators.

Resumen:

Even in one space dimension, the numerical approximation of hyperbolic conservation laws is a challenging task due to the complex structure of the Riemann problem introducing solutions composed of smooth regions separated by discontinuities. Steady states of hyperbolic conservation laws, however, involve either constant solutions, or constant solutions separated by isolated discontinuities (shocks or contact discontinuities). When an additional forcing term is included in the PDE, one speaks of hyperbolic balance laws. In this case, the structure of the solutions becomes even richer.

Indeed, due to the presence of the additional source term, the PDE may exhibit families of non-trivial steady equilibria, which may involve complex variations of the variables as well as discontinuities. Many such states have physical relevance, and in several applications, one is confronted with the need of studying the evolution of very small perturbations of one of such equilibrium states is necessary. Examples can be found both in one and several space dimensions in coastal engineering (e.g. tsunamis, bore propagation in rivers), aerospace and propulsion (e.g. small perturbations of stationary flows in nozzles), astrophysics and large scale geophysical (e.g. small perturbations of vortical flows with gravitational effects).

From the numerical point of view, the approximation of constants is the usual, most natural consistency condition. This means that any perturbation of a constant state can be easily studied numerically, with some classical accuracy and resolution constraints. Exact consistency with non-constant states is, however, not trivial. This has motivated over the last roughly 40 or 50 years the research on numerical methods capable of preserving certain steady state equilibria. This property has been referred to as C-property, of well balanced, or quite simply ‘steady state preserving’ property.

In this short course, we will review the issue of preserving steady states in one and multiple space dimensions. We will show how this notion can be linked to geometrical, and implicit high order ODE integrations in one space dimension.

The extension to multiple space dimensions naturally leads to notion of compatible spatial approximations, capable of preserving differential constraints.

Some examples of numerical methods and some numerical results are used to illustrate the theoretical notions.